S.204 Nr. 14

$\vec{a} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} -2 \\ 4 \\ -1 \end{pmatrix}$

$$\left| 3 \cdot (\vec{a} - \vec{b}) - (\vec{b} - 2\vec{a}) \right| =$$

$$= \left| 3 \cdot \left(\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \\ -1 \end{pmatrix} \right) - \left(\begin{pmatrix} -2 \\ 4 \\ -1 \end{pmatrix} - 2 \cdot \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} \right) \right|$$

$$= \left| 3 \cdot \begin{pmatrix} 3 \\ -4 \\ 4 \end{pmatrix} - \left(\begin{pmatrix} -2 \\ 4 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 6 \end{pmatrix} \right) \right|$$

$$= \left| \begin{pmatrix} 9 \\ -12 \\ 12 \end{pmatrix} - \begin{pmatrix} -4 \\ 4 \\ -7 \end{pmatrix} \right| = \left| \begin{pmatrix} 13 \\ -16 \\ 19 \end{pmatrix} \right|$$

$$= \sqrt{13^2 + (-16)^2 + 19^2} = \cancel{\sqrt{842}} \; \sqrt{786} \; (\approx 28{,}04)$$

Nr. 16 $\left| \begin{pmatrix} 0 \\ 1 \\ 13 \end{pmatrix} \right| = \sqrt{0^2 + 1^2 + 13^2} = \sqrt{170} \quad (\approx 13{,}04)$ 2

$\left| \begin{pmatrix} -3 \\ 4 \\ 12 \end{pmatrix} \right| = \sqrt{(-3)^2 + 4^2 + 12^2} = \sqrt{169} = 13$ 3

$\left| \begin{pmatrix} -2 \\ 1 \\ 13 \end{pmatrix} \right| = \sqrt{(-2)^2 + 1^2 + 13^2} = \sqrt{174} \quad (\approx 13{,}19)$ 1

Der Vektor $\begin{pmatrix} -2 \\ 1 \\ 13 \end{pmatrix}$ hat den größten Betrag!

Nr. 17 $A(2|2|1) \rightarrow \vec{A} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$

$B(-1|1|1) \Rightarrow \vec{B} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$C(-2|-3|-2) \rightarrow \vec{C} = \begin{pmatrix} -2 \\ -3 \\ -2 \end{pmatrix}$

$M(m_1|m_2|m_3) \rightarrow \vec{M} = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}$

$$\overrightarrow{AM} = \vec{M} - \vec{A} = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} m_1 - 2 \\ m_2 - 2 \\ m_3 - 1 \end{pmatrix}$$

$$\overrightarrow{BM} = \begin{pmatrix} m_1 + 1 \\ m_2 - 1 \\ m_3 - 1 \end{pmatrix} \qquad \overrightarrow{CM} = \begin{pmatrix} m_1 + 2 \\ m_2 + 3 \\ m_3 + 2 \end{pmatrix}$$

Bestimme m_1, m_2, m_3 so, dass $|\overrightarrow{AM}| = |\overrightarrow{BM}| = |\overrightarrow{CM}|$

aber auch: $|\overrightarrow{AM}|^2 = |\overrightarrow{BM}|^2 = |\overrightarrow{CM}|^2$

$$|\overrightarrow{AM}|^2 = (m_1 - 2)^2 + (m_2 - 2)^2 + (m_3 - 1)^2 \qquad (\mathrm{I})$$

$$|\overrightarrow{BM}|^2 = (m_1 + 1)^2 + (m_2 - 1)^2 + (m_3 - 1)^2 \qquad (\mathrm{II})$$

$$|\overrightarrow{CM}|^2 = (m_1 + 2)^2 + (m_2 + 3)^2 + (m_3 + 2)^2$$

$(\mathrm{I}) = (\mathrm{II})$ $\quad (m_1 - 2)^2 + (m_2 - 2)^2 + \cancel{(m_3 - 1)^2} = (m_1 + 1)^2 + (m_2 - 1)^2 + \cancel{(m_3 - 1)^2}$

$$(m_1 - 2)^2 + (m_2 - 2)^2 = (m_1 + 1)^2 + (m_2 - 1)^2$$

$$m_1^2 - 4m_1 + 4 + m_2^2 - 4m_2 + 4 = m_1^2 + 2m_1 + 1 + m_2^2 - 2m_2 + 1$$

$$\cancel{m_1^2} + \cancel{m_2^2} - 4m_1 - 4m_2 + 8 = \cancel{m_1^2} + \cancel{m_2^2} + 2m_1 - 2m_2 + 2$$

$$6 = 6m_1 + 2m_2 \quad \Rightarrow \quad 3 = 3m_1 + m_2$$

aus (I) und (II) folgt also $3m_1 + m_2 = 3$

Da es unendlich viele Lösungen gibt wähle z.B.

$m_1 = 2$ und $m_2 = -3$ $\Rightarrow 3 \cdot 2 + (-3) = 3$ ✓

einsetzen in (I) = (III)

$$\Rightarrow (2-2)^2 + (-3-2)^2 + (m_3 \mp 1)^2 = (2+2)^2 + (-3+3)^2 + (m_3+2)^2$$

$$0^2 + (-5)^2 + \cancel{m_3^2} - 2m_3 + 1 = 4^2 + 0^2 + \cancel{m_3^2} + 4m_3 + 4 \quad | +2m_3$$

$$25 + 1 - 16 - 4 = 6m_3$$

$$6 = 6m_3$$

$$\Rightarrow \underline{\underline{m_3 = 1}}$$

Ein mögliches Punkt: $M(2|-3|1)$

gleicher Abstand: $|\overrightarrow{AM}| = |\overrightarrow{BM}| = |\overrightarrow{CM}| = \underline{\underline{5}}$

alternativ: $M^*(3|-6|-\frac{14}{3})$